Temperature fluctuations in the atmospheric boundary layer

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A systematic analysis is made of published measurements of the magnitude of temperature fluctuations in the atmospheric boundary layer. These cover a wide range of height, wind speed, and thermal stratification. Within appropriate ranges of the variables, there is evidence for the existence of a dominantly forced convection régime, and also one wherein the predictions of the similarity theory of free convection are fairly closely approached. Subject to the limitations set by the recording systems used, regression relations are obtained for the magnitude of the fluctuations in terms of height and vertical temperature gradient or heat flux.

1. Introduction

Townsend (1959) has made a study of the temperature fluctuations in a laboratory chamber, heated at the bottom. Defining a length scale z_0, in terms of the upward heat flux H and thermometric conductivity κ, by

$$z_0 = \left(\frac{\rho c_p T \kappa^3}{gH}\right)^{\frac{1}{4}},$$

Townsend finds that the root mean square temperature fluctuation, σ_T, rises to a maximum value near $z/z_0 = 3$, and from $z/z_0 = 6$ to 90 decreases with height according to the relation

$$\frac{\sigma_T}{T\theta_0} = 1{\cdot}6\left(\frac{z}{z_0}\right)^{-0{\cdot}6} \tag{1}$$

θ_0 is a scale of logarithmic temperature defined by

$$\theta_0^4 = \frac{1}{\kappa g}\left(\frac{H}{\rho c_p T}\right)^3,$$

and the other symbols have their usual meaning.

We shall here, similarly, seek to relate appropriately normalized forms of σ_T and z in an analysis of the considerable body of published measurements in the atmospheric boundary layer, under conditions of both heating and cooling from below. However, the typical values of H by day are from 10 to 20 mW cm^{-2}, at times larger, whence $z_0 \approx 0{\cdot}12$ cm. Townsend's results therefore correspond to the lowest 10 cm or so of the atmosphere and, for this reason alone, need not resemble the behaviour at the commonly used heights between 25 cm and 30 m.

Moreover, and more basically, the typical atmospheric condition includes some mean horizontal motion, implying the existence at sufficiently low levels of a régime of effectively forced convection. With the near-boundary condition so radically altered, there is less reason to expect the explicit appearance of molecular quantities, such as κ, in the relationships. Rather, the main purpose of the analysis is changed to that of determining the depth of the effectively forced convection layer and also, by day, how rapidly the transition takes place to a dominantly free convection régime which generally exists at sufficient height. This is truly free convection in that the motions transporting the heat originate from buoyancy, so that wind speed and wind shear no longer appear in the relationships: nevertheless, the presence of these elements, and the difference in conditions below the free convection layer, may give rise to properties substantially different from those of free convection in otherwise perfectly still air.

An analysis of measurements of H (Priestley 1955; Taylor 1956) has suggested that for practical purposes one extreme form of relationship gives way to the other within a rather narrow (twofold or so) range in heights, and there is some evidence that the same holds for the form of the temperature-height profile (Webb 1958). We do not necessarily expect the same limits to apply for the transition of σ_T, since the spectra are known to be dissimilar in that fluctuations of the higher frequencies contribute relatively more to σ_T than to H.

2. Data

Measurements of σ_T were made by Swinbank (1955) at heights from 1·5 to 30 m, over a wide range of degrees of unstable thermal stratification, but avoiding the lightest winds. The limiting factor in frequency response was the recording galvanometer of 2·3 second free period. Taylor (1956, 1958) took further measurements using essentially the same equipment and height range, but selecting the conditions of strongest heating with light wind, generally less than 2 m sec^{-1} at 2 m height. Cramer, Gill and Record of the Massachusetts Institute of Technology measured σ_T during the Great Plains Turbulence Field Programme (Lettau & Davidson 1957) from 1·5 to 12 m, using a galvanometer of 1·1 second free period. Though non-selective in intent, this material (hereafter referred to as M.I.T. data) was in fact mainly confined to conditions of strong wind, averaging 7–8 m sec^{-1} at 2 m during the daytime. Where these sets of data overlapped, no significant separation was found between them, and accordingly the whole is treated herein as one more or less homogeneous body.

When the responsiveness of the overall sensing-recording system is substantially increased, fluctuations of higher frequency are detected, and a correspondingly larger value of σ_T is obtained. Concurrently with the M.I.T. measurements, Stewart and Kassander of Iowa State College (I.S.C. hereafter) recorded σ_T from 0·25 to 8 m height on magnetic tape with a modulator designed to yield a flat amplitude versus frequency response characteristic from zero to 15 c/s (Lettau & Davidson, pp. 206–15). The sensing device was a bead thermistor whose lag time was found to vary in an approximately exponential manner between 0·5 sec at zero wind speed and 0·1 sec in a 5 m sec^{-1} wind. This latter speed was approached throughout and usually exceeded (except at the lower levels during some of the night-time runs). For reasons stated above, the analysis of I.S.C. data has been kept distinct from the remainder.

Details of the procedures used in analysis are described in the Appendix.

3. Variation of σ_T with height in unstable stratification

In the case of potential temperature decreasing with height, it proves useful, as a preliminary analysis, to consider purely the variation of σ_T with height without requiring σ_T to be normalized. We seek to fit a relation of the form

$$\sigma_T \propto z^{-p}. \tag{2}$$

The simultaneous or near-simultaneous measurement of σ_T at any pair of heights will then determine a single value of p (which will be presumed to refer to the geometric mean of the two heights). In this way, over 400 individual values of p were obtained from the I.S.C. data and over 200 from the other material.

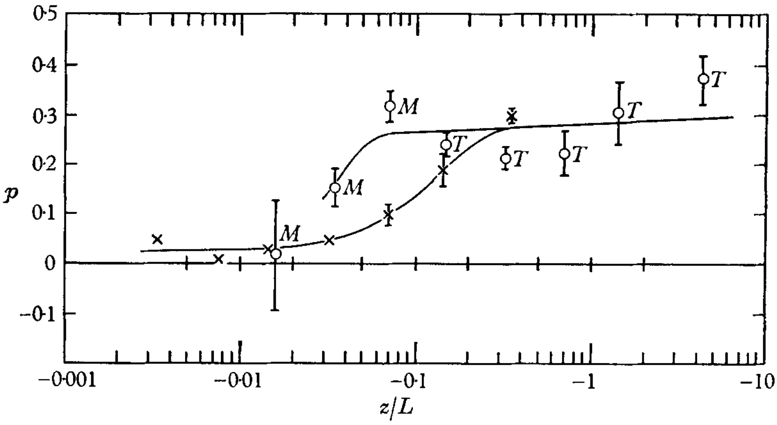

FIGURE 1. Index p as a function of z/L. Marks on the $p = 0$ line show the ranges into which values are grouped. × denotes I.S.C., ○ other data (T = Taylor, M = M.I.T. preponderating). Where no standard error lengths are indicated, this was less than 0·01.

p may be expected to depend on the relative strength of mechanical and convective processes and hence on the ratio z/L, where L is the Obukhov length defined as

$$L = -\frac{c_p \rho T u_*^3}{kgH}. \tag{3}$$

u_* is the friction velocity $(\tau/\rho)^{\frac{1}{2}}$, τ the shearing stress, and k von Karman's constant. The values of p have accordingly been grouped in ranges of z/L, and the mean of each group and its standard deviation are shown in figure 1. The symbols M and T denote that either the M.I.T. or the Taylor data contributed the majority of determinations in the group indicated; in no case did the Swinbank data contribute more than a minority.

In interpretation, we must note that $p = 0$ in forced convection while in free convection, subject to the principles of similarity with the assumption that molecular effects are not explicitly important, $p = \frac{1}{3}$ (Priestley 1954). In the I.S.C. data, mechanically generated fluctuations appear to be dominant up to

$z/|L| \approx 0{\cdot}05$. The régime is close enough to fully forced convection to expect useful working relationships to be obtainable therefrom, though it may be more strictly correct to speak of dominantly forced or quasi-forced convection, corresponding, for example, to the log-linear wind or temperature profile form of Monin & Obukhov (1954) at these small values of $z/|L|$. Above $z/|L| \approx 0{\cdot}05$, there is a relatively rapid change until, shortly above $0{\cdot}1$, the results become indistinguishable from those obtained with the less responsive equipment. These latter, on the other hand, display from as low as $z/|L| = 0{\cdot}05$ a rather strong tendency for dominance by buoyancy-induced effects. What is here perhaps no more than an indication that the transition is largely concentrated within the range 0·02–0·05 becomes stronger when set against earlier, and almost entirely independent, evidence of the transition in the régime of heat flux within the same limits.

4. Absolute value of σ_T in unstable stratification

The absolute analysis of σ_T has been carried out within the framework of the relation

$$\sigma_T/(z\theta')_{z_1} = \phi(z, L), \tag{4}$$

where ϕ is the function which requires to be discovered. θ' is written for $|\partial\theta/\partial z|$, the magnitude of the potential temperature gradient, and z_1 is a fixed height at which θ' was relatively accurately known in all instances, 1·5 m in fact being selected. Ideally it would have been preferable to divide σ_T by $z\theta'$ measured at the same height, or at some height which was a constant multiple or fraction χ of L or, equivalently to the latter, divide it by

$$T_* = H/(\rho c_p k u_*).$$

In any of these alternative analyses the right-hand side would be expected, on similarity theory, to be a function of the ratio z/L rather than of the two variables separately. For reasons appertaining to the data, as discussed briefly in the Appendix, these forms of analysis were not practicable without introducing undesirable corruption. However, the limiting forms of such relationships can be readily derived from the limiting forms of (4) and will be given below.

The results are displayed in figure 2 as a diagram of $\log\{\sigma_T/(z\theta')_{1\cdot5}\}$ against $\log z$, the data being divided into groups with similar values of L. As is foreshadowed in figure 1 and known from other evidence (see e.g. Priestley 1959), the forced fluctuations are of generally shorter period than the free fluctuations, and the more responsive I.S.C. system will record much of the former which the other data will miss. The two sets of data may be clearly distinguished in figure 2, and will be discussed separately in what follows.

The following extreme forms of (4) may readily be verified:

(i) When z and z_1 both lie within the forced convection régime,

$$\sigma_T/(z\theta')_{z_1} = \text{constant}, a.$$

(ii) When z lies in, but z_1 above, the forced convection régime,

$$\sigma_T/(z\theta')_{z_1} = \text{constant}, a'.$$

a' will depend on z_1/L but will always be greater than a.

(iii) When z and z_1 both lie within the free convection régime then, on the similarity theory,

$$\sigma_T/(z\theta')_{z_1} = b(z/z_1)^{-\frac{1}{3}},$$

where b is a constant.

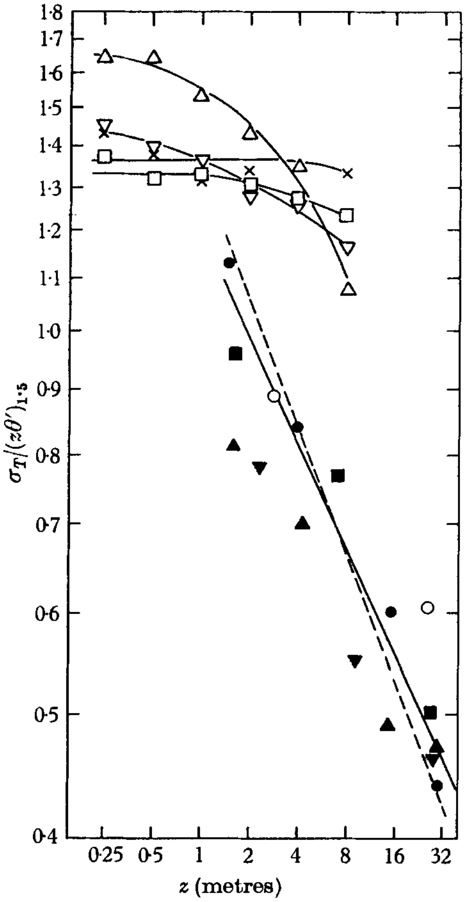

FIGURE 2. Normalized temperature fluctuations as a function of z and L. Designation of points, for I.S.C. data, is as follows: △, $|L| \leqslant 60$ m (average 45 m); ▽, $60 < |L| \leqslant 100$; □, $100 < |L| \leqslant 150$; ×, $150 < |L|$ (average 225 m). For other data: ●, $|L| \leqslant 15$ m (average 8·3 m); ○, $15 < |L| \leqslant 30$; ▲, $30 < |L| \leqslant 60$; ▼, $60 < |L| \leqslant 100$; ■, $100 < |L|$ (average 210 m). Each point represents the mean of at least seven individual determinations, the average being eight for the I.S.C. data (186 determinations) and thirteen for the less homogeneous material of the other data (202 determinations). The full and broken straight regression lines correspond to equations (6) and (7), respectively, of the text: other lines are drawn by eye to fit the four categories of the I.S.C. data.

(iv) When z lies in, but z_1 below, the free convection régime,

$$\sigma_T/(z\theta')_{z_1} = b'(z/z_1)^{-\frac{1}{3}},$$

where b' depends on z_1/L but will always be greater than b.

The upper set of curves in figure 2, corresponding to the I.S.C. data, shows separation of the profiles according to the magnitude of L, and the ranges within which the predictions (i) and (ii) are approached serve to confirm $z/|L| \approx 0{\cdot}02$ as effectively the limit of fully forced convection. Any conclusions as to numerical magnitudes are subject to the limitation imposed by the response times of the recording instruments, but with this reservation the relation for forced convection may be written as

$$\sigma_T = 1{\cdot}4z\,|\partial\theta/\partial z|. \tag{5}$$

The need for a standard height has disappeared because of the constancy of $z\theta'$ through the layer in question. Using the familiar equation $H = \rho c_p K\,|\partial\theta/\partial z|$, where $K = ku_* z$, equivalent forms of (5) are seen to be

$$\sigma_T/T_* = \sigma_T/(z\theta')_{\chi L} = 1{\cdot}4,$$

so long as the level χL is in the same régime.

The points from the other data in figure 2 show no systematic separation according to the magnitude of L. This in itself is an indication of dominance by one or other of the two extreme régimes and the pattern of results, backed by what has gone before, points to free convection in this instance. The two mechanisms are of course to a large extent coexistent and forced fluctuations will occur in the overlapping region, and at even larger values of $z/|L|$; but the less responsive equipment fails to register their finer structure. Nor is it proved that all the free fluctuations are fully registered, though it appears likely on internal and other evidence that this is approximately true. In making any further comparison between the two sets of points, note should be taken of the range of L as well as of z.

For the 'other' data, in the lower part of figure 2, regression coefficients were calculated between $\log\{\sigma_T/(z\theta')_{1\cdot5}\}$ and $\log z$ excluding, for reasons evident from figure 1, those points for which $z/|L| < 0{\cdot}05$, and weighting the remainder according to the number of individual contributing values. The result obtained was

$$\sigma_T/(z\theta')_{1\cdot5} = 1{\cdot}19z^{-0\cdot28} = 1{\cdot}07(z/1{\cdot}5)^{-0\cdot28}, \tag{6}$$

where z is measured in metres. This is shown by the full line in the diagram. The broken line shows the best fit when the index is constrained to the value $-\frac{1}{3}$, the regression then being

$$\sigma_T/(z\theta')_{1\cdot5} = 1{\cdot}34z^{-\frac{1}{3}} = 1{\cdot}20(z/1{\cdot}5)^{-\frac{1}{3}}. \tag{7}$$

In the similarity theory of free convection, the temperature profile is given by $\theta' \propto z^{-\frac{4}{3}}$, and where this holds (7) is equivalent to a more basic form of relationship

$$\sigma_T = 1{\cdot}2z\,|\partial\theta/\partial z|, \tag{8}$$

in which the need for a standard height as intermediary has again been eliminated. The formal similarity between (5) and (8) is interesting. Whether it portrays a physical similarity or conceals a physical difference is possibly a matter for debate, but it certainly suggests that an analysis carried through in terms of $\sigma_T/z\theta'$, instead of $\sigma_T/(z\theta')_{z_1}$, had it been practicable, would have proved considerably less illuminating.

An alternative form of (8), which portrays the height-variation explicitly, may be obtained from the heat flux relation of similarity theory, namely

$$H = h\rho c_p(g/T)^{\frac{1}{2}}z^2\,|\partial\theta/\partial z|^{\frac{3}{2}},$$

where h is the free convection constant whose value is about 0·9 (Priestley 1959). This leads immediately to

$$\sigma_T/T_* = 1{\cdot}2k^{\frac{4}{3}}h^{-\frac{2}{3}}|z/L|^{-\frac{1}{3}} \approx 0{\cdot}4\,|z/L|^{-\frac{1}{3}}.$$

5. Inversion conditions

The analysis of σ_T in stable stratifications is confined to the I.S.C. and M.I.T. material, Taylor's studies having included no such cases and Swinbank's only a few of more uncertain quality. Winds ranged between 3 and 9·5 m sec^{-1} at 8 m, and between 1 and 5·5 m sec^{-1} at 0·5 m.

The means for obtaining L, as described in the Appendix for unstable conditions, are no longer available. Instead, we shall make direct use of Ri′ as tabulated by Lettau & Davidson and defined as

$$\mathrm{Ri}' = \frac{\Sigma\,\mathrm{Ri}(z_i)}{\Sigma z_i},$$

where $\mathrm{Ri}(z_i)$ was obtained from the profile measurements at a number (up to 5) of heights z_i more or less proportionately spaced from 0·8 to 8 m. Since $\mathrm{Ri}' = 1/L$ in fully forced convection, and may be expected to behave in a closely sympathetic fashion elsewhere, $z\mathrm{Ri}'$ is a valid substitute for z/L in the search for universal relationships.

The I.S.C. data was first grouped in ranges of Ri′ and it was found that, in each group, σ_T increased slowly with height. The rate of increase corresponded to a p in (2) of between $-0{\cdot}01$ and $-0{\cdot}05$ resembling, with change of sign, the behaviour towards the left in figure 1. It was accordingly considered suitable to make the absolute analysis of σ_T in the same terms as that of the previous section, save that the fixed height z_1 was here chosen as 65 cm in order to permit $(z\theta')_{z_1}$ to be read off directly from the published data (see Appendix). The slow variation of σ_T with height permitted values to be estimated where these were occasionally missing from one or two levels, so rendering the material homogeneous.

The average (geometric mean) values of $\sigma_T/(z\theta')_{0\cdot65}$ were then obtained in ranges of $z\mathrm{Ri}'$ and the results are shown in table 1, I.S.C. and M.I.T. data being treated separately. The first value of the I.S.C. data appears anomalous, probably because both σ_T and θ' are relatively most inaccurate in this range, and the use of occasional spuriously low values of θ' may have biased the quotient in the sense indicated. Otherwise, below $z\mathrm{Ri}' \approx 0{\cdot}1$ there is no significant departure from the overall mean value of 1·33 and, by analogy with the unstable case, we may infer the existence of a dominantly forced convection régime wherein

$$\sigma_T = 1{\cdot}33z\left|\frac{\partial\theta}{\partial z}\right|. \tag{9}$$

Within the accuracy of determination there is no significant difference between the constants in (9) and (5).

Above $z\mathrm{Ri}' \approx 0{\cdot}1$ the suppressive influence of stability enters, and this begins to take effect on the longer-period fluctuations (M.I.T. data) at much smaller values of $z\mathrm{Ri}'$. Here the decrease to about one-third in a 20-fold range in $z\mathrm{Ri}'$ appears sharper than the corresponding decrease in the unstable condition. Moreover, the two are to be further distinguished by the fact that $z\theta'$ typically increases with height in the more stable conditions, so that a relationship of the form (8) is here no longer even remotely applicable.

Range	$z\mathrm{Ri}' \leqslant 0{\cdot}002$	$0{\cdot}002 < z\mathrm{Ri}' \leqslant 0{\cdot}005$	$0{\cdot}005 < z\mathrm{Ri}' \leqslant 0{\cdot}01$	$0{\cdot}01 < z\mathrm{Ri}' \leqslant 0{\cdot}02$	$0{\cdot}02 < z\mathrm{Ri}' \leqslant 0{\cdot}05$	$0{\cdot}05 < z\mathrm{Ri}' \leqslant 0{\cdot}1$	$0{\cdot}1 < z\mathrm{Ri}' \leqslant 0{\cdot}2$	$0{\cdot}2 < z\mathrm{Ri}'$
I.S.C. data								
No. of observations	21	36	35	35	41	27	10	5
Av. $\sigma_T/(z\theta')_{0{\cdot}65}$	1·73	1·43	1·25	1·31	1·34	1·26	1·04	0·87
M.I.T. data								
No. of observations	—	—	13	24	42	33	15	13
Av. $\sigma_T/(z\theta')_{0{\cdot}65}$	—	—	0·96	0·95	0·77	0·63	0·51	0·30

TABLE 1. $\sigma_T/(z\theta')_{0{\cdot}65}$ as a function of $z\,\mathrm{Ri}'$.

6. Conclusions

It has been seen that a considerable body of data on temperature fluctuations is available from the atmospheric boundary layer, but its definitive interpretation is limited by the responsiveness of the recording systems used. Subject to this limitation, quite well-defined regression relations have been obtained. From the more responsive equipment, the total intensity σ_T follows a régime of dominantly forced convection up to $z/|L| = 0{\cdot}05$ to $0{\cdot}1$ on either side of neutral conditions. Above these levels, the effects of stratification make themselves felt quite rapidly. However, the longer-period fluctuations, which alone are recorded by the less responsive equipment, are subject to strong thermal influence above $z/|L| \approx 0{\cdot}02$ on either side of neutral conditions, and in unstable conditions above $z/|L| \approx 0{\cdot}05$ obey a relationship not very different from the predictions of the similarity theory of free convection. These in part consolidate the views previously arrived at from the study of heat-flux measurements in unstable conditions. To the extent that further interest exists in the fluctuations *per se*, a more complete exploration is called for with equipment responding to periods even shorter than 0·1 sec, combined with some form of spectral analysis.

Appendix. Details of the analysis

The analysis was restricted to occasions when conditions were steady, as far as could be determined from the data. In particular, runs made at or near the times of cross-over between stable and unstable stratifications were excluded whenever there was evidence that one or other of these conditions were not maintained throughout the relevant range of heights.

Before analysis, certain amendments were made to the data as published. Two values of σ_T published by Taylor (1958) were subsequently found to be in error: in run 2 of 1 February 1956, σ_T was 0·60 and 0·55 °C at 1·5 and 4 m, respectively.

The gradient of potential temperature at 1·5 m, reported in Swinbank's and Taylor's papers, had been derived by an objective smoothing (as described in the former) of the measured data at $\frac{1}{2}$, 1, 2 and 8 m. In a few instances where this might have introduced some error owing to the incidence of vanishing gradients below 8 m in strongly unstable situations (Webb 1958), the gradient was recalculated using only $\frac{1}{2}$, 1 and 2 m. For the Great Plains data, $z\theta'_{1\cdot5}$ was taken as the value given in Table 7.3*b* of Lettau & Davidson. It is seen that this

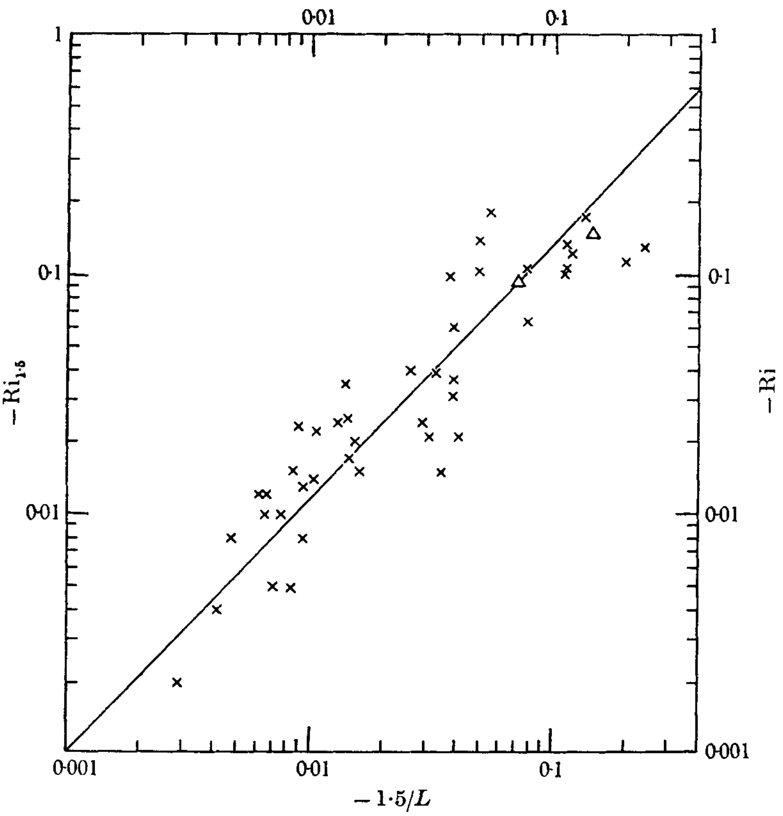

FIGURE 3. $\mathrm{Ri}_{1\cdot5}$ versus $1\cdot5/L$ from Swinbank's measurements (L in metres). Triangles denote values obtained by Rider.

is actually a weighted mean of gradients between 82 and 39 cm and between 160 and 40 cm, and applies to a level of about 65 cm. In general, this identification was justified because of the strong winds and rather modest temperature gradients, but in the one case (1235 on 8 September 1953) when conditions were sufficiently unstable to invalidate it, $z\theta'_{1\cdot5}$ was interpolated from the original temperature data using all measurements between 0·5 and 4 m inclusive.

Acceptable measured values of L were available only in Swinbank's material. Here, although both H and τ were biased to some extent by the responsiveness of the recorder, the resulting underestimation was common to both so that L should be rather more accurate than its numerator or denominator separately. Flux measurements at higher levels were notably less reliable than at the lower levels (4 m and below) and the latter only were used, these being assumed to apply to a proximate high level run provided the interval was less than $\frac{1}{2}$ hour and the $\mathrm{Ri}_{1\cdot 5}$ did not change by more than $\frac{1}{3}$ of the prior value.† Where these conditions were not satisfied, the high level σ_T was not included in the material.

For the Great Plains and Taylor's data, in unstable conditions, L was obtained as follows. Figure 3 shows values of $-\mathrm{Ri}_{1\cdot 5}$ reported by Swinbank plotted against the corresponding value of $-1\cdot 5/L$ obtained from the low-level H and τ measurements. Two points are also shown as obtained by Rider (1954). This allows the straight line as drawn, or its prolongation, to be used as a basis for obtaining L from $\mathrm{Ri}_{1\cdot 5}$, which was always available either directly or from Ri' as tabulated by Lettau & Davidson. It is to be noted that L has been used only as a grouping parameter, or as divisor of z, for quantities which are never more than slowly varying functions of z. Modest errors in L will therefore not substantially affect the conclusions.

Coming finally to the choice of equation (4) as the framework of the absolute analysis it may be seen that T_*, as an alternative divisor of σ_T, could have been inferred with an accuracy comparable with that of L. However, errors acceptable in the divisor of z would have been much more serious in the divisor of the less accurately known, and much slower varying, quantity σ_T. Of other possible divisors, $z\theta'$ at the same height as σ_T was not always available, and its individual measurement would anyway be subject to increasing relative inaccuracy at the greater heights. Similar considerations forbade the use of $z\theta'$ at χL, however χ might be chosen, since $|L|$ ranged from 2 m to several hundred metres.

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† The same criterion for steadiness was applied to Swinbank's data for accepting values σ_T from two successive runs for comparison through equation (2).